

Spin-orbit coupling in interacting quasi-one-dimensional electron systems

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We present a new model for the study of spin-orbit coupling in interacting quasi-one-dimensional systems and solve it exactly to find the spectral properties of such systems. We show that the combination of spin-orbit coupling and electron-electron interactions results in: the replacement of separate spin and charge excitations with two new kinds of bosonic mixed-spin-charge excitation, and a characteristic modification of the spectral function and single-particle density of states. Our results show how manipulation of the spin-orbit coupling, with external electric fields, can be used for the experimental determination of microscopic interaction parameters in quantum wires.

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In contemporary condensed matter physics there exists a great variety of electron systems that can be considered as quasi-one-dimensional (Q1D). Among them are conducting polymers [1], carbon nanotubes [2], and semiconductor heterostructures [3]. They combine the richness of observable physical properties with the possibility of finding exact solutions to non-trivial interacting problems. The major theoretical advance in this field was the formulation and solution of the Tomonaga-Luttinger model [4], which revealed features that are generic to many interacting Q1D electron systems, such as separation of charge and spin degrees of freedom and the anomalous scaling of correlation functions [5].

The very existence of Q1D systems (or quantum wires) arises essentially from confining electrical forces, which prevent the lateral motion of electrons. The accompanying electric fields necessarily give rise to a coupling between the spin and orbital degrees of freedom of propagating electrons — the spin-orbit (SO) interaction. Although this interaction is relativistic in nature, it nevertheless results in a significant modification to the band structure of Q1D systems (see, e.g., experimental [6] and theoretical [7] papers). In strictly 1D and 2D systems, the role of the SO interaction basically reduces to a “horizontal” (in momentum space) splitting of the energy branches corresponding to spin-up and spin-down electron states. In contrast to this, the existence of quantised transverse energy subbands in quantum wires brings about an additional important manifestation of the SO coupling: the absence of a vertical symmetry axis for each spin branch (see Fig. 1 and Ref. [8]). This absence results in a breakdown of chiral symmetry so that the electron Fermi velocities become dependent on the direction of motion. This effect monotonically increases as the SO coupling is enhanced. As applied to interacting systems, the absence of chiral symmetry raises two fundamental questions: how does the system respond to the asymmetry of the single-particle spectrum; and what is the fate of the spin-charge separation in the presence of

the SO coupling? It is the aim of this Letter, to establish a realistic model for the SO coupling in interacting 1D systems which answers these questions, and to provide experiment with a set of quantitative predictions which will allow their verification.

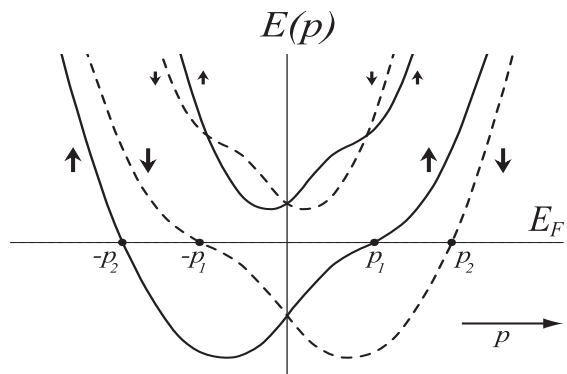


FIG. 1. Single-electron energy spectrum in a quantum wire with spin-orbit interaction. The dotted line is the Fermi energy level $E = E_F$ and $\pm p_{1,2}$ are the Fermi momenta of respective groups of electrons.

In constructing a Hamiltonian for such a system, we consider the case where the Fermi energy E_F is sufficiently small, such that only the lowest energy subband in a quantum wire is partly filled, while all the others are empty. This regime has proven to be the richest in non-trivial experimental results [2,9], and the SO effects are expected [8] to be most pronounced here. As a natural way of capturing the essential physics in a quantum wire, we suggest the use of a modification to the Tomonaga-Luttinger model that takes into account the asymmetric single-particle spectrum in Fig. 1. Namely, we take the Hamiltonian to have the form $H = H_0 + H_{int}$, where H_0 describes the linearized spectrum,

$$H_0 = -iv_1 \int dx \left(\psi_{R,\uparrow}^\dagger \partial_x \psi_{R,\uparrow} - \psi_{L,\downarrow}^\dagger \partial_x \psi_{L,\downarrow} \right)$$

$$-iv_2 \int dx \left(\psi_{R,\downarrow}^\dagger \partial_x \psi_{R,\downarrow} - \psi_{L,\uparrow}^\dagger \partial_x \psi_{L,\uparrow} \right), \quad (1)$$

and H_{int} is responsible for the EE interaction:

$$H_{int} = g_2 \int dx \psi_{R,\uparrow}^\dagger \psi_{R,\uparrow} \psi_{L,\downarrow}^\dagger \psi_{L,\downarrow} + g_4 \int dx \psi_{R,\uparrow}^\dagger \psi_{R,\uparrow} \psi_{L,\downarrow}^\dagger \psi_{R,\downarrow} + (R \leftrightarrow L). \quad (2)$$

The operators $\psi_{r,s}(x)$ ($r = R, L$; $s = \uparrow, \downarrow$) annihilate spin-up (\uparrow) and spin-down (\downarrow) electrons near the right (R) and left (L) Fermi points. In Eq. (1) the electron velocities $v_{1,2} = \partial E(p_{1,2})/\partial p$ are derived from the dispersion law in Fig. 1. In general, $v_1 \neq v_2$ as long as the spin-orbit interaction is finite [8].

In the interaction Hamiltonian (2) we include only forward scattering contributions, while the backward and Umklapp scattering are left out. Neglecting the Umklapp scattering is known [5] to be safe if energy bands are far from being half-filled, which is exactly the case, e.g., in quantum wires patterned in semiconductor heterostructures [3]. As regards the backscattering, it can be shown that, similar to the case of zero SO coupling [5], it renormalizes to zero for repulsive interactions and therefore can also be omitted. In this letter, we concentrate on the major tendencies arising from the SO coupling in interacting electron systems, therefore we assume momentum-independent interaction potentials $g_2 > 0$ and $g_4 > 0$ in Eq. (2), which corresponds to a well-screened repulsion between electrons.

The Hamiltonian (1), (2) is reminiscent of that for the multi-component Tomonaga-Luttinger model [10,5] which consists of mutually interacting Luttinger liquids with different Fermi velocities. However, in contrast to our case, the model [10,5] assumes that each liquid has a *symmetric* single-electron spectrum (as occurs, for example, with Zeeman splitting [11]) and therefore, as we will show, it describes qualitatively different physical behaviour.

We study the interacting model (1), (2) with the help of the bosonization technique [12]. Within this formalism, fermionic operators $\psi_{r,s}(x)$ are represented as $\psi_{r,s}(x) \sim \exp(-i\Phi_{r,s})$, where the operators $\Phi_{r,s}(x)$ are linear combinations of so-called phase fields $\varphi_{\rho(\sigma)}(x)$ and $\Pi_{\rho(\sigma)}(x)$ that obey the bosonic canonical commutation relations (see Refs. [12] for details). In terms of these phase fields, the Hamiltonian (1), (2) takes the bosonized form:

$$H = \frac{1}{2\pi} \int dx \left\{ v_\rho K_\rho (\pi \Pi_\rho)^2 + \frac{v_\rho}{K_\rho} (\partial_x \varphi_\rho)^2 + v_\sigma K_\sigma (\pi \Pi_\sigma)^2 + \frac{v_\sigma}{K_\sigma} (\partial_x \varphi_\sigma)^2 + \delta v [(\pi \Pi_\rho) \partial_x \varphi_\sigma + (\pi \Pi_\sigma) \partial_x \varphi_\rho] \right\}, \quad (3)$$

where

$$v_0 = (v_1 + v_2)/2, \quad \delta v = v_2 - v_1 \quad (4)$$

and $v_{\rho(\sigma)} = [(v_0 \pm g_4/2\pi)^2 - (g_2/2\pi)^2]^{1/2}$ and $K_{\rho(\sigma)} = [(2\pi v_0 \mp g_2 \pm g_4)/(2\pi v_0 \pm g_2 \pm g_4)]^{1/2}$. The first (second) line in Eq. (3) describes free propagation of charge (spin) density wave with the velocity $v_{\rho(\sigma)}$ and “stiffness” coefficient $K_{\rho(\sigma)}$. The third line in Eq. (3) is proportional to the velocity difference δv and therefore represents the strength of the SO interaction. In the Tomonaga-Luttinger model, where $v_1 = v_2$, this term is absent and this results in a decoupling of the charge and spin degrees of freedom: the so-called *spin-charge separation* [5]. This decoupling inhibits the existence of quasi-particles with spin 1/2 and charge $-e$, the basic excitations of a Fermi liquid, and gives rise to a different state of matter, the Luttinger liquid, which has bosonic excitations in the form of independent spin and charge density waves.

As the SO interaction is switched on, i.e. at $\delta v \neq 0$, the third term in Eq. (3) starts to affect the dynamics of the system. It couples together the ρ - and σ -fields and thereby *destroys* this spin-charge separation. A similar effect is also found in a spinful Luttinger model in a magnetic field [11]. In our case, the asymmetry of the single-electron spectrum in Fig. 1 results in a different mechanism for the violation of spin-charge separation from Ref. [11]. We stress however, that our model is exactly solvable since the Hamiltonian (1), (2) is quadratic in the phase fields. Here we choose to solve it within the functional integration formalism in imaginary (Matsubara) time [13].

The most profound effects of the SO coupling on interacting Q1D systems can be seen in the behaviour of single-particle characteristics, such as the spectral function $\rho_r(q, \omega)$,

$$\rho_r(q, \omega) = -\frac{1}{\pi} \sum_s \text{Im} G_{r,s}^{(ret)}(q, \omega), \quad r = R, L, \quad (5)$$

and the density of states $N(\omega) = (1/2\pi) \sum_r \int dq \rho_r(q, \omega)$. Here q and ω are the momentum and energy of elementary excitations and $G_{r,s}^{(ret)}(q, \omega)$ is the Fourier transform of the retarded Green function

$$G_{r,s}^{(ret)}(x, t) \equiv i\theta(t) \langle \{ \psi_{r,s}(x, t), \psi_{r,s}^\dagger(0, 0) \} \rangle = -i\theta(t) [G_{r,s}(x, t) + G_{r,s}(-x, -t)], \quad (6)$$

where $\theta(t)$ is the Heaviside step function and $G_{r,s}(x, t) = \langle \psi_{r,s}(x, \tau) \psi_{r,s}^\dagger(0, 0) \rangle|_{\tau \rightarrow it}$ with τ being the Matsubara time. Leaving detailed calculations to a regular article, we give here an exact expression for $G_{r,s}(x, t)$ at $T = 0$:

$$G_{r,s}(x, t) = \frac{\exp(ip_{r,s}x)}{2\pi\Lambda} \left(\frac{\Lambda}{\Lambda + i(u_1t - x)} \right)^{\theta_1^+} \left(\frac{\Lambda}{\Lambda + i(u_1t + x)} \right)^{\theta_1^-} \left(\frac{\Lambda}{\Lambda + i(u_2t - x)} \right)^{\theta_2^+} \left(\frac{\Lambda}{\Lambda + i(u_2t + x)} \right)^{\theta_2^-}. \quad (7)$$

A small (of the order of the lattice parameter) length scale Λ defines ultraviolet cut-off [12]. From Eq. (7) it follows that although the SO coupling destroys the spin-charge separation, it nevertheless preserves the anomalous scaling of correlation functions. At $\delta v = 0$, Eq. (7) coincides with the well-known expression [5]. The exponents θ_i^\pm ($i = 1, 2$) are determined by values of $v_{\rho(\sigma)}$, $K_{\rho(\sigma)}$, δv and depend on the chiral r and spin s indices. For arbitrary interaction constants g_2 and g_4 , the expressions for θ_i^\pm are rather lengthy and here we give them for the realistic case of equal interaction strength between all the spectral branches, i.e. for $g_2 = g_4 \equiv (\pi v_0)\beta$:

$$\theta_i^\pm(r, s) = (-1)^i \xi_i^\pm(r, s) \eta_i / 2(\eta_1^2 - \eta_2^2), \quad (8)$$

$$\xi_i^\pm(r, s) = \pm r \left[(1 + r s \epsilon / 2)^2 / \eta_i - \eta_i \right] - (1 - r s \epsilon / 2) + \left[(1 + r s \epsilon / 2)(1 - \epsilon^2 / 4) - \beta^2 / 2 \right] / \eta_i^2.$$

Here $\epsilon = \delta v / v_0$, and on the r.h.s. of the last equation $r = \pm 1$ for right (left) spectrum branches and $s = \pm 1$ for spin-up (spin-down) electrons.

The quantities $\eta_{1,2} \equiv \eta_{1,2}(\epsilon)$ defined by

$$\eta_{1,2}^2 \equiv (u_{1,2} / v_0)^2 = 1 + \epsilon^2 / 4 \mp \sqrt{\beta^2 + \epsilon^2} \quad (9)$$

are dimensionless velocities of the new *independent* bosonic excitations that replace the spin and charge density waves, respectively, as the single-electron spectrum becomes asymmetric ($\epsilon \neq 0$). Each of these new excitations are superpositions of the old ones, they carry *both* charge and spin, and have a sound-like spectrum $\omega = u_{1,2}q$. For $\epsilon = 0$, we have $u_{1,2} = v_{\sigma(\rho)}$ and return to the spin-charge separation. Eq. (9) demonstrates that increasing ϵ at constant β pushes u_1 and u_2 away from v_σ and v_ρ as well as from each other: one of the excitations monotonically accelerates (u_2 grows) while the other monotonically slows down (u_1 decreases). This effect becomes more pronounced in systems with stronger EE interaction. Since the parameter ϵ is controlled by the SO interaction and therefore by the lateral electrical confinement, the strength of this confinement must affect drastically the dynamics of elementary excitations in quantum wires. Further increasing ϵ eventually results in a vanishing of the velocity u_1 at the “critical” point $\epsilon_0 = 2\sqrt{1 - \beta}$ and in a possible phase transition accompanied by phase separation [5]. This phenomenon is beyond the scope of this Letter.

Eq. (7) allows one to calculate the spectral function $\rho_r(q, \omega)$ which can be obtained in magnetotunneling experiments [14,15]. Fig. 2 shows the spectral function $\rho_R(q, \omega)$ for right-moving electrons in the range of energies ω where all singularities of $\rho_R(q, \omega)$ are located.

At each value of $\epsilon < \epsilon_0$, the spectral function has two singular points corresponding to the bosonic excitations with velocities u_1 and u_2 [see Eq. (9)]. In full accordance with Eq. (9), the distance between the singular points grows with ϵ . The singularities of $\rho_R(q, \omega)$ are governed by power laws with exponents depending on $v_{\rho(\sigma)}$, $K_{\rho(\sigma)}$, and ϵ . The dependence on ϵ is such that one of the singularities becomes sharper while the other becomes smoother as ϵ grows. It is noteworthy that the dependence of ρ_R on q at fixed ω is very similar to that shown in Fig. 2 and exhibits the same tendencies as a function of ϵ .

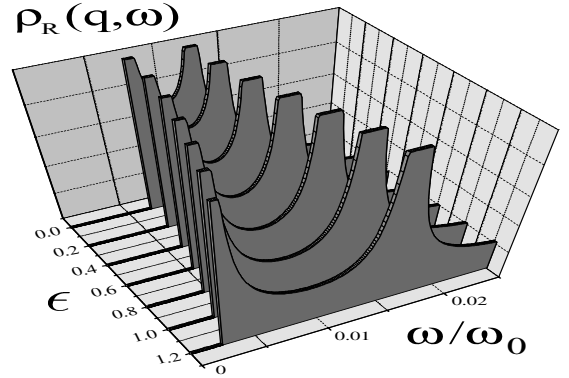


FIG. 2. Spectral function $\rho_R(q, \omega)$ (in arbitrary units) for a fixed $q = 0.01\Lambda^{-1}$ and several values of ϵ and for $\beta = 0.5$ ($\omega_0 \equiv v_0/\Lambda$). Note that $\rho_L(q, \omega) = \rho_R(q, -\omega)$.

From Eqs. (5), (6), and (7) one can also calculate the single-particle density of states $N(\omega)$. This quantity may be obtained in tunneling measurements since the tunneling current, e.g., between a wide metal and an interacting quantum wire is proportional to $N(eV)$, where V is an applied voltage (see Ref. [16]). It can also be determined in angle-integrated photoemission experiments [17]. For $\omega \rightarrow 0$ we obtain the following behaviour:

$$\frac{N(\omega)}{N_0} = \frac{1}{4} \sum_{r,s} \frac{(\omega/\omega_0)^{\theta_1 + \theta_2 - 1}}{\eta_1^{\theta_1} \eta_2^{\theta_2} \Gamma(\theta_1 + \theta_2)}, \quad \theta_i = \theta_i^+ + \theta_i^-. \quad (10)$$

Here $\omega_0 = v_0/\Lambda$ is the natural energy unit of the order of the Fermi energy, $N_0 = 2/\pi v_0$ is the constant density of states at $\beta = \epsilon = 0$, and $\Gamma(x)$ is the Gamma function.

Fig. 3a demonstrates the effect of the EE interaction on the density of states $N(\omega)$ at a fixed strength of the SO coupling. At zero EE interaction ($\beta = 0$), the function $N(\omega)$ is a constant, $N(\omega)/N_0 = 1/(1 - \epsilon^2/4)$. As the EE interaction is switched on ($\beta \neq 0$), it starts to affect the formation of the lowest lying excitations and $N(\omega)$ takes on a power-law behaviour in the vicinity of

$\omega = 0$. The width of this power-law interval becomes progressively larger as β grows and leads to a monotonic suppression of the density of states. This effect is also present for zero SO coupling [5].

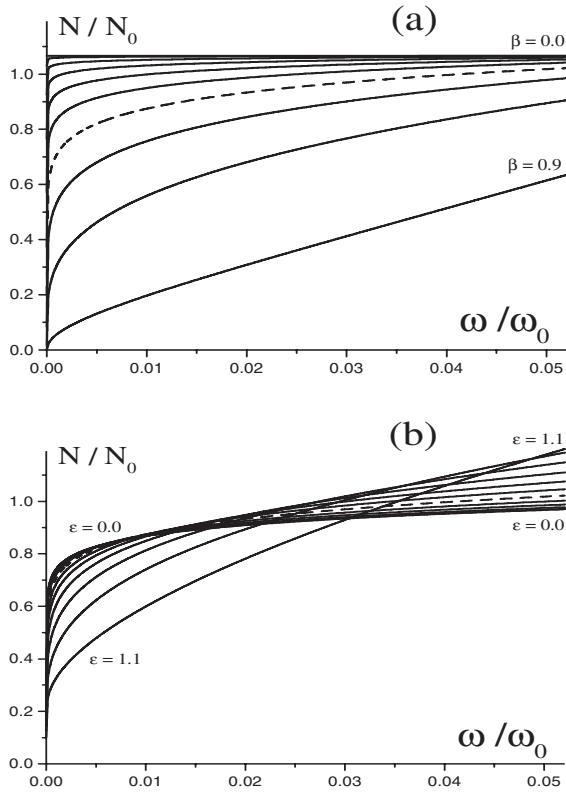


FIG. 3. Normalised single-electron density of states: (a) $\epsilon = 0.5$ and $\beta = 0.0, 0.1, \dots, 0.9$; (b) $\beta = 0.6$ and $\epsilon = 0.0, 0.1, \dots, 1.1$. The dashed curves correspond to $\epsilon = 0.5$, $\beta = 0.6$.

Fig. 3b shows the evolution of the function $N(\omega)$ for fixed β as the SO coupling is varied. For large values of ω , where the role of EE interactions is not significant, the magnitude of $N(\omega)$ increases as ϵ grows. However, at small values of ω , where the nature of the elementary excitations is essentially dictated by EE interactions, the effect of the SO coupling on $N(\omega)$ is qualitatively the same as that of the EE interaction, that is, increasing ϵ leads to a suppression of the density of states. We emphasize that in the standard multi-component Tomonaga-Luttinger model [10,5] as well as in spin-polarized Luttinger liquids [11], there is no interval of ω where the density of states is suppressed by increasing the velocity difference between spectral branches. The existence of such an interval is a unique manifestation of the SO coupling in quantum wires.

The characteristic dependence of the spectral function $\rho(q, \omega)$ and the density of states $N(\omega)$ on the microscopic parameter ϵ , for fixed β , should enable one to extract both quantities from experiment and thus determine how

strong the EE and SO interactions are. Such dependencies can be obtained experimentally by changing the SO coupling directly by varying an electric field perpendicular to a quantum wire (quantum-well field) as it was done, e.g., in Ref. [18].

Theoretical calculations [8] of the electron bandstructure modified by the SO coupling indicate that the value of ϵ in typical Q1D semiconductor systems should be $\sim 0.1 - 0.2$ and appears sufficiently large to observe the principal tendencies in the behaviour of $\rho(q, \omega)$ and $N(\omega)$ caused by the SO coupling.

In conclusion, we have formulated and solved analytically the problem of the interplay between EE and SO interactions in Q1D electron systems and found that: (i) the spin-charge separation breaks down while the anomalous scaling of correlation functions is preserved; (ii) the single-particle characteristics, such as the spectral function and the density of states, are essentially modified and controlled by the strength of the SO coupling; (iii) varying the SO coupling with the external electric field can be used to extract the microscopic parameters of quantum wires.

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